The right gyrator trims the fat off active filters

Replacing inductors with gyrators creates almost perfect filters

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☐ Analog filters exhibiting nearly ideal performance can be built around a gyrator—if the right configuration of this active circuit is used. In effect, the gyrator makes a capacitance behave like an inductor, freeing the filter of the problems plaguing conventional inductors, like large size, low Q, winding capacitance, nonlinearity, and magnetic susceptibility.

Yet most designers look upon the gyrator as an idealistic circuit with a "peculiar" behavior that puts it out of touch with practical applications. This attitude completely ignores its power. Unlike other active-filter circuits, the gyrator permits the designer to take advantage of the large body of data and techniques already developed for passive LC filters. He can start with a passive prototype circuit and then replace each inductor with a gyrator, substantially reducing filter size and weight for frequencies up to about 50 kilohertz.

Fortunately, too, there is one gyrator realization that works superbly. Not all of them do—in the past, different versions have suffered from drawbacks like instability, poor control of loss, sensitivity to component matching, and even excessive complexity. But the preferred version is simple and stable and simulates a high-quality inductor, permitting very high-performance filters to be realized. In addition, this gyrator, unlike other active-filter circuits, preserves the most significant advantage of coupled LC networks—their inherently low sensitivity to changes in component values (see "The strength of LC filters," p. 116).

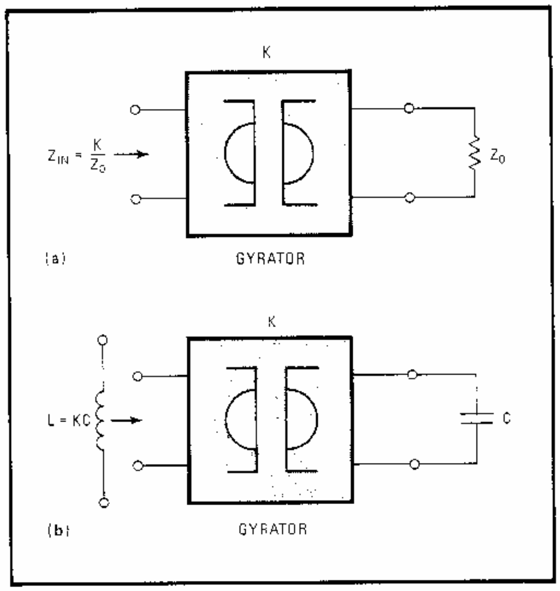
Understanding the gyrator

Basically, the gyrator is a lossless two-port circuit (Fig. 1a) that inverts a load impedance. When used with a high-Q capacitor (Fig. 1b), it simulates the virtual characteristics of a high-Q inductor. The preferred realization for the gyrator requires only two amplifiers and five impedances, as shown in Fig. 2 for both the general impedance representation (a) and the practical RC implementation (b). In the latter case, the circuit simulates an inductor having a value of KC, where K is a constant determined by the resistors:

$$K = R_1R_3R_5/R_2$$

At first glance, this gyrator's need for two amplifiers may seem a disadvantage. However, consider the major drawback of most single-amplifier resonators. They generally require an amplifier having a gain in excess of Q^2; and those that do not usually are extremely sensitive to passive-element variations. On the other hand, the gyrator does not require a high-gain amplifier—in fact, stable Qs of better than 1,000 may be obtained with only 40 decibels of gain. Furthermore, unlike other active-filter circuits, the gyrator is remarkably insensitive to any amplifier parameter, so it may be built with garden-variety devices, even quad chips, as long as they are unity-gain-stable amplifiers.

Additionally, with the gyrator, amplifier phase shift enhances Q, rather than diminishing it as in other active-



1. Ideally. Coupled LC filters simulated with gyrators have characteristics approaching the ideal. In effect, the gyrator is a lossless two-port circuit (a) that inverts a load impedance. With a capacitive load impedance, the circuit (b) simulates a high-quality inductor.

The strength of LC filters

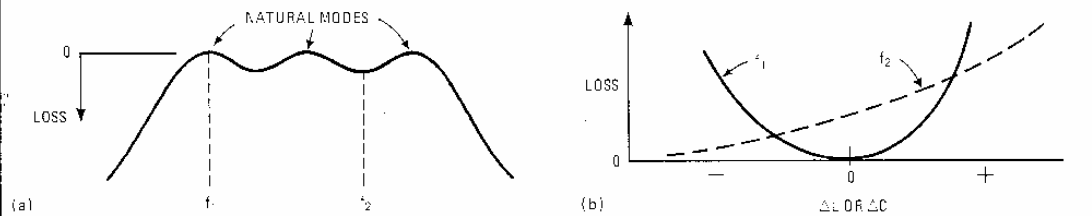
In theory, coupled LC filters have the lowest sensitivity to component variations. These doubly terminated reactive two-ports produce a frequency response by reflecting power back to the source in the stopbands. In the passband, power transfer is maximum at natural modes (a) determined by the filter's transfer function.

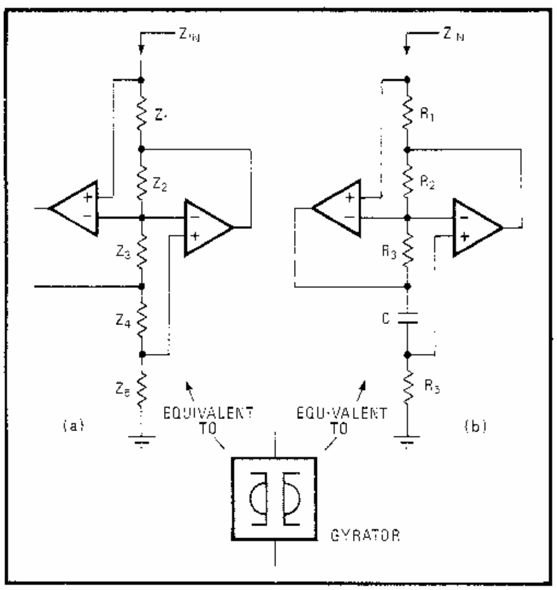
In the real world, a change in any inductor or capacitor making up the filter can only cause a loss in the load power—down from the maximum. This power loss at one of the natural modes (f_1) increases monotonically (b), while at frequencies other than the natural modes, such as f_2, a small loss exists because of the ripple caused by reflected power. At any point within the passband, then, the change in loss has a well-behaved and slowly varying characteristic that follows the changes of any inductor or capacitor in the network.

Since no one inductor or capacitor determines a natural mode, a change in any single inductance or capacitance can only partially affect the shift in a natural-mode frequency. It follows that coupled LC filters are inherently insensitive to changes in component values. Similarly, if all of the inductors in the filter are replaced by gyrators, this insensitivity to component variations does not change, since the gyrator is an active circuit and adds no dissipative elements to the filter.

Other active-filter circuits, however, like the biquad and state-variable or universal active filters, are developed from the state equations describing a second-order transfer function. Since these circuits duplicate the properties of a second-order (one complex pole pair) LC filter, their component sensitivity is still the same as for coupled second-order networks. On the other hand, extension to higher orders requires factorization of the transfer function into biquadratic (second-order plus lower-order) factors, each of which specifies a separate Q and natural-mode frequency. The desired transfer function is then realized by cascading biquadratic stages. Since these stages are uncoupled, changes can easily occur in the amplitude or frequency of the simulated modes, making the high-order filters built this way sensitive to component variations.

An alternative is the design approach called leap-frog. It implements the state equations of the prototype LC filter directly, using integrators and summing amplifiers. But though the resulting filter does have about the same low sensitivity as the equivalent coupled LC network, the final circuit can become very complex for high-order functions.





2. Realistically. Preferred gyrator realization (a) requires two amplifiers and five impedances. In practical RC implementation (b), impedance Z_4 is a capacitor, and the other impedances are resistors. The gyrator may also be represented by the special symbol shown here.

filter circuits. At the ideal phase shift of 90°, the Q of the simulated inductor is approximately equal to that of the capacitor being used. If the phase shift is greater than 90°—which is usually the case—the Q becomes even higher.

Needless to say, the inductor the gyrator simulates is not perfect—the gyrator can be no better than the resistors and capacitors with which it is built. Of the two, resistors are less worrisome, for tin-oxide, metal-film, and thin-film types all perform acceptably. Capacitors, on the other hand, are the weakest link in the gyrator circuit, and there are usually two or more of them per complex pole pair. They impose the first limitation—maximum Q—in any realization, and their capacitance may change a lot with temperature. The table reviews the important characteristics of a variety of capacitor types. Generally, NPO ceramic devices are least affected by temperature, whereas polypropylene units achieve the highest Q.

Creating a floating gyrator

One seeming limitation of the gyrator is that it is grounded at one end. But the floating inductor often needed in a filter can be simulated successfully—for example, by connecting two grounded gyrators. However, this does not necessarily mean that an extra gyrator is required for every floating inductor in a passive LC filter. Figure 3a shows two gyrators sharing the bottom resistor, R_5, at opposite ends, so as to simulate a single floating inductor. This resistor is described in the gyration constant:

$$L = \frac{R_1R_3R_5}{R_2}C$$

The equation may be rewritten as:

$$L = \frac{R_1R_3C}{R_2}R_5$$

which says that the simulated inductance is directly proportional to the value of R_5. Therefore, if R_5 in fact becomes a loaded port for the gyrator, the simulated inductance will depend on the value of resistance connected to that port.

A cursory examination of the preferred gyrator realization (in Fig. 2b) will reveal a corollary to the above relationship. Between the top of R_1 (the input port) and the bottom of C, a voltage null exists because of the amplifiers' input connections. Therefore, the port to which R_5 is connected has the same voltage as the input port (although the currents are not the same, otherwise an apparent inductance could not exist). As a result, the input impedance, Z_{IN}, is also proportional to resistor R_5. This resistor could even be a network of resistors to describe the topological connections, of, say, a T or pi network of inductors, as indicated in Fig. 3b.

Designing another floating gyrator

Indirectly, a floating inductor may be achieved in another way—one that opens up other possibilities for converting a passive LC filter to its active equivalent. When a second capacitor is added to the gyrator and one resistor removed, the circuit becomes a functionally dependent negative resistor (FDNR)—an element that appears to be a negative resistance that decreases in value with frequency.

Consider the transfer function for the basic impedance converter of Fig. 2a:

$$Z_{IN} = (Z_1Z_3Z_5)/(Z_2Z_4)$$

If Z_1 and Z_3 are capacitors and the other impedances are resistors, then:

$$Z_1 = Z_3 = 1/sC$$

where $s = j\omega$. The input impedance can then be written as:

$$Z_{IN} = \text{FDNR} = \frac{1}{s^2}\frac{R_5}{C^2R_2R_4} = -\frac{1}{\omega^2}\frac{R_5}{C^2R_2R_4}$$

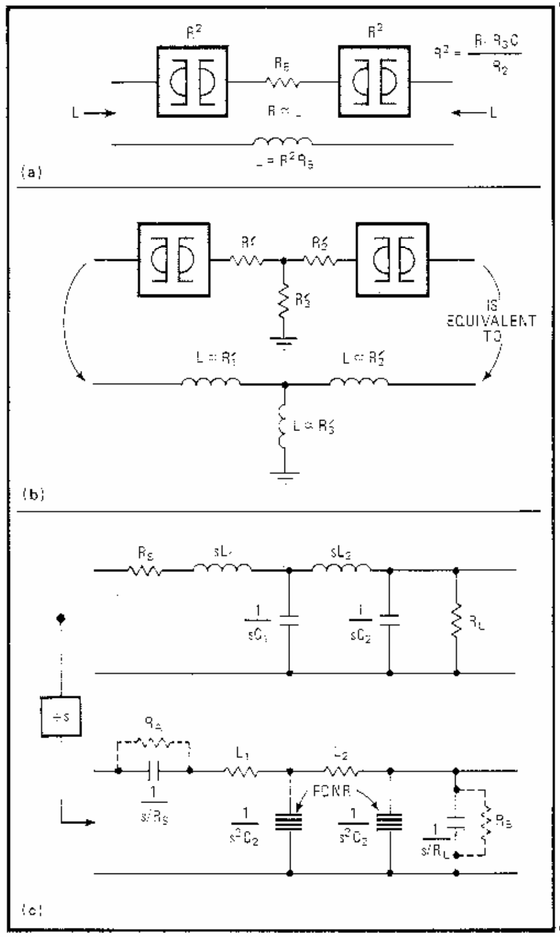
This element may be used to solve the problem of simulating floating inductors in low-pass filters. The technique is simple—just divide all elements by s, which is the complex variable, and then replace the $1/s^2$ terms with an FDNR, as shown in Fig. 3c. The floating inductors become resistors having a value of L. Resistor R_A and resistor R_B simply provide bias and response for the circuit at dc.

Although the gyrator is not the easiest circuit to understand, designing with it is really not that hard. Suppose the requirement is for a double-tuned bandpass

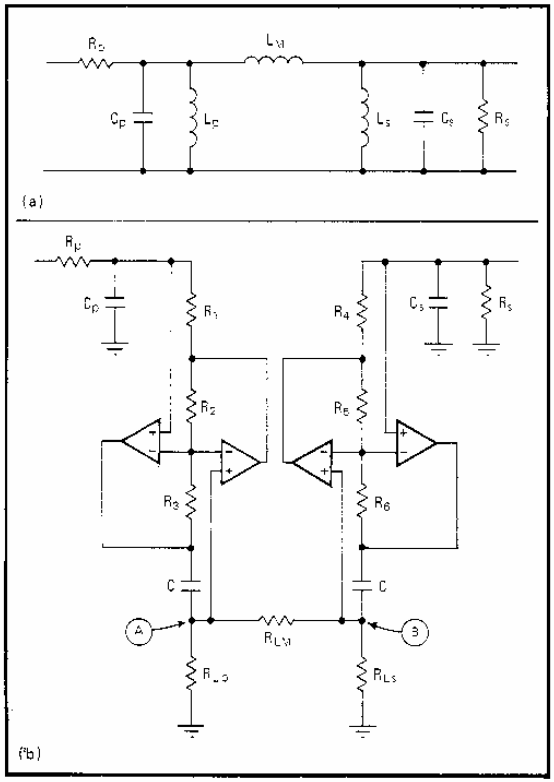
CAPACITOR CHARACTERISTICS

Type	Q (at 1 kHz)	Temperature coefficient (ppm/°C)	Temperature range (°C)
Mica	600	1 to −70	−55 to +125
Polystyrene	2,000	−150 ±50	−55 to +85
NPO ceramic	1,500	±30	−55 to +125
Polypropylene	3,000	−115	−55 to +125
Glass	1,500	+140 ±25	−55 to +125
Polycarbonate**	500	≈ 50*	−40 to +100
Mylar**	100	Large	−55 to +85
Polyester**	100	−160*	−40 to +100
Porcelain	2,500	±25	−55 to +125

*0°C to 50°C **Q and C nonlinear functions of frequency and function



3. Floating. Lower terminal of basic gyrator (Fig. 2b) is grounded. To float the circuit, two gyrators may share the same resistor (a), or a network of resistors (b). Floating inductors may also be simulated (c) with functionally dependent negative resistors (FDNRs).



4. Bandpass filter. Double-tuned bandpass filter of (a) may be built with two gyrators, as in (b). Sharing resistor R_{LM}, the grounded gyrators simulate the pi network of inductors in the passive version. All of the amplifiers may be general-purpose devices.

filter, like the one drawn in Fig. 4a. The procedure is straightforward. First compile the design data required:

e_r, the desired passband ripple, expressed in peak-to-peak decibels;

f_o, the center frequency in hertz;

f_r, the ripple bandwidth in Hz;

C, the capacitance value for both C_p and C_s in farads;

r, the termination ratio of R_p/R_s.

Next, calculate these variables:

$$A = (10^{e_r/20})^{-1}$$

$$q = \left[\frac{2(1 - A^2)^{1/2} + r + (1/r)}{2 - 2(1-A^2)^{1/2}}\right]^{1/2}$$

$$Q' = f_0/f_r$$
$$\alpha = 2(1-A^2)^{1/2}$$
$$X = [\alpha(1 + q^2)]^{1/2}$$
$$Q = Q'X$$

$$X' = \left\{\frac{\alpha}{2}(1 + q^2)\left[1 + \frac{[\alpha^2 + 4(10^{6/20} - 1)]^{1/2}}{\alpha^2}\right]\right\}^{1/2}$$

Then compute the design results:

$$R_s = \frac{Q}{\omega_0 C r^{1/2}}$$

$$R_p = R_s r$$

$$L_M = \frac{R_s r^{1/2}}{\omega_0 q}$$

$$L_p = L_s = \frac{L_M}{\omega_0^2 C L_M - 1}$$

$$G_0 = \frac{1}{r^{1/2}}\left[\frac{q}{1 + q^2}\right]$$

where G_0 is the midband gain.

$$BW(3\ dB) = \frac{X'}{X} f_r$$

where BW(3 dB) is the 3-dB bandwidth.

Building a double-tuned filter

Two gyrators replace the pi network of inductors, as shown in Fig. 4b. Now assign the same capacitance value for C as that chosen for C_p and C_s. The amplifiers may be a quad-type 4136, since choosing such a device ensures a good match between the amplifiers for optimum gyrator performance. Next, determine the gyrator resistances from:

$$L_p = (R_1 R_3 R_{Lp} C)/R_2$$

It is good practice to maintain the resistances at about the same value, so assume:

$$R_1 = R_2 = R_3 = R_{Lp} = R$$

Then:

$$R^2 = L_p/C$$

The value of R should be large enough to minimize amplifier loading and slightly smaller than the amplifier differential input impedance. For convenience, choose the closest standard value for R and let:

$$R = R_1 = R_2 = R_3$$

Then compute:

$$R_{Lp} = L_p/CR$$

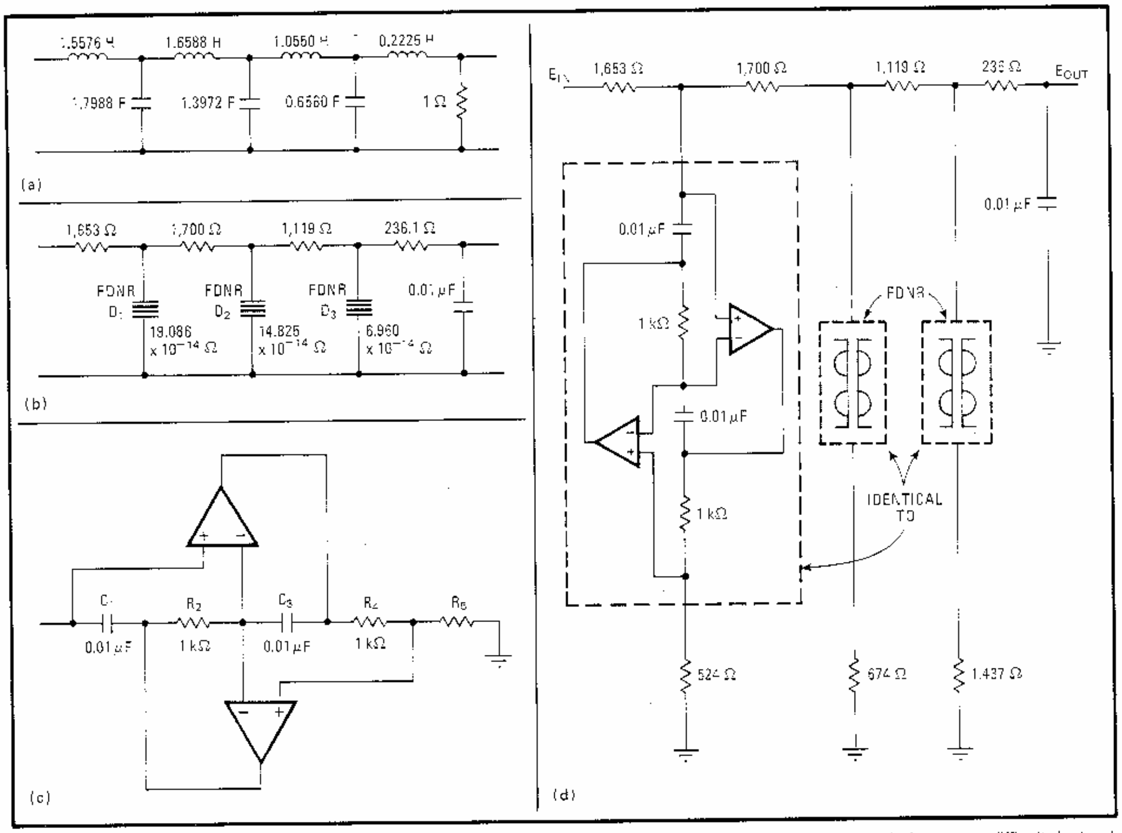
Again select a nearby standard value for R_{Lp} and scale R_{LM}:

$$R_{LM}/R_{Lp} = L_M/L_p$$

This ratio must be maintained, since it determines the coupling in the circuit. All of the component values are now known. The tolerances for C may be ±5% and ±1% for R, although the tolerances for $R_{1\text{-}6}$ could be looser as long as the devices' environmental characteristics are acceptable.

Tuning the filter

Tuning is simple. Overall Q has already been determined by the value of R_{LM}. Using point A as the output, the primary (left-hand) gyrator is set by shorting point B to ground and adjusting R_1 or R_2 to achieve resonance at the center frequency. Reducing R_1 (by shunting it) will decrease L_p, raising the resonant frequency. The reverse holds for R_2. Tune for 0° phase shift relative to the



5. Aliasing filter. Number of floating inductors in low-pass aliasing filter (a) makes implementation with grounded gyrators difficult. Instead, FDNRs may be used (b), each of which requires two capacitors (c). The final circuit (d) employs only three of these FDNRs.

source. Next, disconnect point B from ground, and tune R_4 or R_5 for 90° phase shift (at the center frequency) at the output, or point A, which is equivalent. (Tuning with a Lissajous circle can provide accuracy to better than 2°.)

A second example demonstrates how to design with FDNRs. The requirement is for a low-pass aliasing filter—a seventh-order Butterworth circuit having a 1-dB corner at 15 kilohertz.

Using functionally dependent negative resistors

A prototype circuit (Fig. 5a) is first obtained from one of the standard tables in existing literature. Here, the number of floating inductors prevents easy implementation with the grounded gyrators, and the best approach is to use their close relative—the FDNR.

To convert the LC prototype circuit to an FDNR realization, first normalize the corner frequency to 1 radian per second. Next scale the circuit for frequency by dividing the inductors and capacitors by $2\pi(15\text{ kHz})$, and then scale the impedances for a convenient capacitor value, say 0.01 microfarad or $1/(0.01 \times 10^{-6})$ ohms, in which case multiply the inductances by 10^8 and divide the capacitances by 10^8. Finally, dividing all of the network impedances by the complex variable, s, results in the FDNR realization of Fig. 5b.

Each FDNR is actually the basic gyrator configuration, but with two capacitors (Fig. 5c) instead of just one. In this example, all of the resistors, except R_5, are set equal to 1 kilohm, and the two capacitors to 0.01 microfarad each. Then R_5 may be computed from:

$$R_5 = (R_2R_4C_1C_3)/D$$

where D is the impedance value of the FDNR. Therefore, for D_1, R_5 = 524 ohms; for D_2, R_5 = 674 ohms, and for D_3, R_5 = 1.437 kilohms.

Figure 5d shows the final circuit, in which all seven capacitors have the same value, an essential point in low-cost design. The circuit may be built with 1% resistors having values closest to those computed and with NPO ceramic capacitors screened to tolerances of ±1%. Again, garden-variety amplifiers, like the 4136 quad, perform well enough for the purpose. □

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