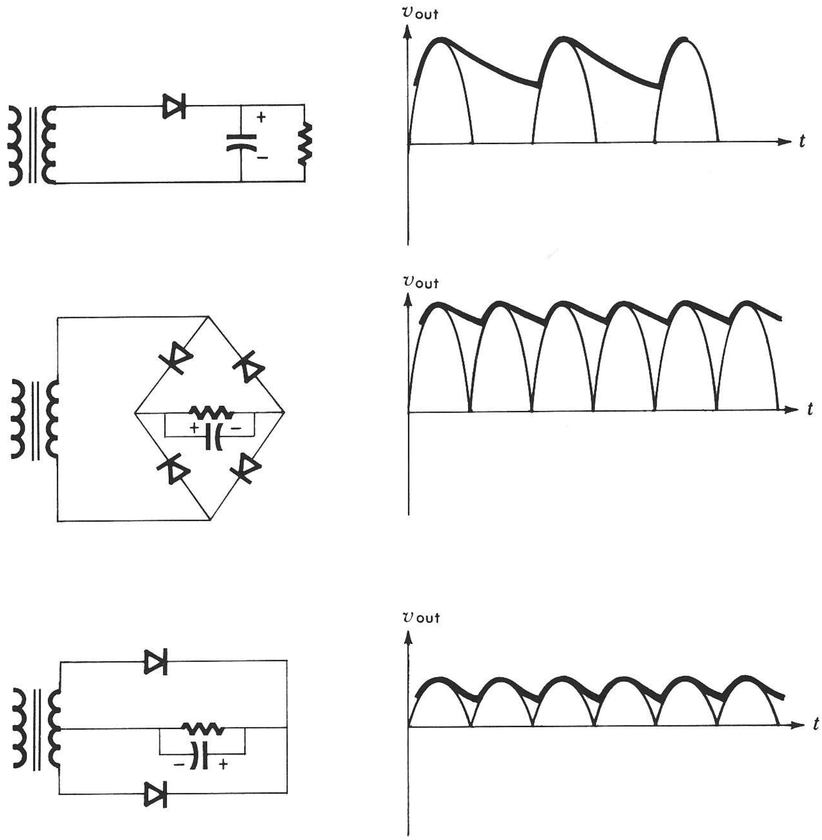

FIGURE 4-5 *Comparison between the outputs of the three different capacitor-filtered power supplies.*

A QUANTITATIVE STUDY

Let us begin our quantitative study by first considering a numerical description of the ripple content of the output waveshape.

A generally accepted figure of merit regarding ripple is the *ripple factor*, which is defined as

$$\text{Ripple factor} \equiv k_r \equiv \frac{\text{rms of ripple}}{\text{average output level}}$$

Although it is not exactly true, the ripple is assumed to be a sine wave. It is the rms value of this fictitious sine wave which is indicated in the above formula for k_r. The rms value is obtained by noting the peak-to-peak value of the ripple and then converting it into the equivalent rms that such peak-to-peak readings would indicate had the ripple been a sine wave

$$\text{rms} = \frac{\text{peak-to-peak}}{2\sqrt{2}}$$

Expressed as a percent, the ripple factor is called the *percent ripple.*

The average output level or the dc level, as it may be called, can be approximated closely by taking it as the midpoint between the upper and the lower peaks of the ripple.

Illustrative Problem 4-1

Determine the ripple factor and the percent ripple of the waveshape shown in Fig. Prob. 4-1.

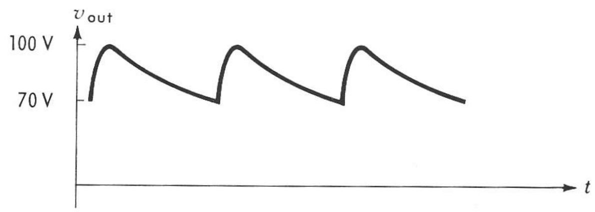

FIGURE PROB. 4-1

SOLUTION

$$\text{Ripple peak-to-peak} = 100 - 70 = 30 \text{ volts}$$

$$\text{Ripple peak} = \frac{\text{peak-to-peak}}{2} = 15 \text{ volts}$$

$$\text{rms of sine-wave equivalent} = \frac{\text{peak}}{\sqrt{2}} = \frac{15}{\sqrt{2}} = 10.6 \text{ volts}$$

$$\text{dc level} = 70 + \frac{100 - 70}{2} = 70 + \frac{30}{2} = 85 \text{ volts}$$

$$k_r = \frac{\text{rms ripple}}{\text{dc level}} = \frac{10.6}{85} = 0.125$$

$k_r = 0.125$
% ripple = 12.5%

HOW TO CHOOSE THE CAPACITOR

A key question as yet unanswered is: How do we choose the proper sized capacitor to function as a filter in a power supply? Obviously

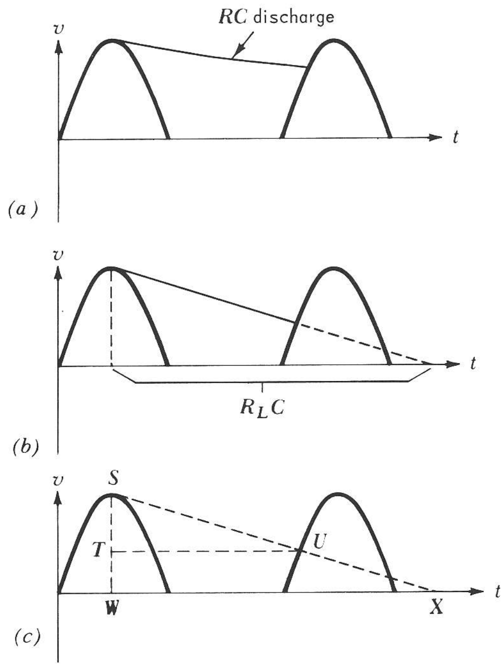

FIGURE 4-6 *The waveshapes and constructions used in the development of the formula for C_{min}.*

one answer is to use the largest capacitor available. We cannot eliminate all the ripple completely, and the larger the capacitor, the more ripple removed. Then we must ask: What is the minimum sized capacitor we can get away with using, in order to limit the ripple factor to the maximum value permitted by the design requirements?

Let us begin our study of this problem by analyzing the waveshapes shown in Fig. 4-6. We can make the approximation that the *RC* discharge portion of the output curve is a straight line, based on the assumption that the time constant involved is very long compared to that portion of the discharge curve which is part of the output waveshape. With this as an assumption, the extension of the discharge curve will intercept the horizontal time axis after a portion of time equal to one *RC* time constant has elapsed, as noted in Fig. 4-6*b*. Recall that the slope of the *RC* discharge curve, just as the discharge begins, is such as to attempt to discharge the capacitor within one time constant.

Another approximation we must make is that the time elapsed between the peak of the output wave and the point at which the discharge curve intercepts the next half-cycle of the sine wave is equal to the fraction 1 over frequency of the ripple, or the ripple period. Again, this is only an approximation because it ignores that portion of time in which the sine-wave segment of the output waveshape occurs. This does not generate too great an error because as long as we keep the ripple content small, the sine wave segment is part of the output waveshape for only a very small portion of the period. This will provide us with a reasonably close approximation to the size capacitor we are seeking. In Fig. 4-6*c* we can make out two similar triangles

$$\triangle STU \approx \triangle SWX$$

Using our knowledge of the relationship which exists between the corresponding sides of similar triangles, we can write

$$\frac{\text{Side } SW}{\text{Side } ST} = \frac{\text{Side } WX}{\text{Side } TU}$$

And since

$$\text{Side } SW = V_{\text{peak out}}$$
$$\text{Side } ST = V_{\text{p-p ripple}}$$
$$\text{Side } WX = R_L C$$
$$\text{Side } TU = \frac{1}{f_{\text{ripple}}}$$

we get

$$\frac{V_{\text{peak out}}}{V_{\text{p-p ripple}}} = \frac{R_L C}{1/f_{\text{ripple}}}$$

Is this equation adequate for determining the proper value of a capacitor? Let us see if we have only one unknown with this one equation. To start with, the frequency of the ripple is known once we know the frequency of the power line and whether we are using a full-wave or a half-wave rectifier. R_L is usually known since this is the resistance of the load being fed.

Once we know the ripple content and the required dc level, we can find $V_{\text{peak out}}$. Again, however, if we assume that the ripple content will be small compared to the dc level, we can approximate $V_{\text{peak out}}$ by $V_{\text{dc out}}$. This simplifies matters greatly since $V_{\text{dc out}}/V_{\text{p-p ripple}}$ can be found directly from k_r, the ripple factor. k_r is usually determined by the type of service for which the supply is intended. This equation is then adequate because we are left with just one unknown. It would be more convenient if we were to swap the ratio of peak output voltage to peak-to-peak voltage for some relationship involving the ripple factor, which we now do.

$$k_r = \frac{V_{\text{rms ripple}}}{V_{\text{dc}}} = \frac{V_{\text{p-p ripple}}/2\sqrt{2}}{V_{\text{dc}}}$$

$$V_{\text{p-p ripple}} = k_r V_{\text{dc}}\ (2\sqrt{2})$$

$$= 2\sqrt{2}k_r V_{\text{dc}}$$

$$\frac{V_{\text{peak out}}}{V_{\text{p-p ripple}}} \approx \frac{V_{\text{dc}}}{V_{\text{p-p ripple}}} \approx \frac{R_L C}{1/f_{\text{ripple}}}$$

$$\frac{V_{\text{dc}}}{2\sqrt{2}k_r V_{\text{dc}}} \approx \frac{R_L C}{1/f_{\text{ripple}}}$$

$$\frac{1}{2\sqrt{2}k_r} \approx R_L C$$

$$\boxed{C_{\text{min}} \approx \frac{1}{2\sqrt{2}k_r R_L f_{\text{ripple}}}}$$

This value of capacitance is the *minimum* value needed in order to limit the ripple to the value set by k_r.

Illustrative Problem 4-2

A full-wave-rectified power supply is to be designed to operate directly from a 120-volt 60-Hz line and supply a 10,000-ohm load. The output voltage is to be approximately 170 volts dc. The percent ripple is to be limited to 5 percent. Determine the minimum value of capacitance which must be placed across the output of the rectifier in order to meet the requirements of the design.

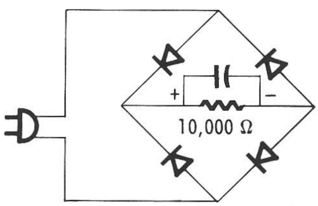

FIGURE PROB. 4-2

SOLUTION

$$C_{\text{min}} = \frac{1}{2\sqrt{2}k_r R_L f_{\text{ripple}}}$$

$$= \frac{1}{2\sqrt{2} \times 0.05 \times 10{,}000 \times 120}$$

$$\boxed{C_{\text{min}} = 5.9\ \mu\text{f}}$$

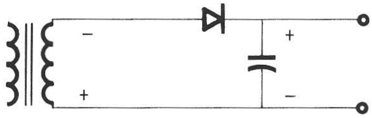

FIGURE 4-7 *Half-wave capacitor-filtered power supply with polarities shown for PIV studies.*

ADDITIONAL STRESSES ON THE DIODE DUE TO THE CAPACITOR FILTER

Let us examine the power-supply circuits to see if additional stresses are placed on the diodes due to the added capacitor filter. First we will study the half-wave circuit shown as Fig. 4-7.

The voltage across the capacitor is equal to the peak voltage of the transformer secondary. This voltage across the capacitor remains constant. When the voltage across the transformer secondary has the polarity as shown in Fig. 4-7 and the peak voltage occurs, we have the greatest PIV because the voltage across the transformer and the voltage across the capacitor are series-aiding, placing a PIV across the diode of $2V_{\text{peak trans sec}}$, twice the peak voltage which occurs across the transformer secondary.

In the case of full-wave rectifiers the PIV situation is the same as that which existed without the capacitor, because we already had to deal with the case of a peak voltage across the output and a peak voltage across the transformer secondary occurring simultaneously in a series-aiding combination.

What about the current-handling requirements of the diode in the capacitor-filtered situation? Obviously it will have to be greater than without the capacitor because the diode is now back-biased for a longer time, and since the average output current is the same as before, greater currents must pass through the diodes because they are conducting for a shorter time span.

Making use of the principle of conservation of electric charge, we see that the total amount of electric charge which is sent to the capacitor during the conduction portion of the cycle must equal the amount of charge the capacitor is feeding the load over the rest of the cycle. Again a few approximations will greatly simplify our analysis. Since the capacitor feeds the load for the greater amount of time, we can make the approximation that the capacitor feeds the load for the entire cycle, and thus we can write that the total amount of electric charge which the capacitor feeds to the load during the entire cycle must be fed to the capacitor during the time that the diode conducts. Since electric charge is equal to current times the amount of time the current is flowing, we can write

$$i_c t_c = I_{\text{dc}} T$$

where i_c = charging current flowing into capacitor
t_c = capacitor charging time
I_{dc} = dc load current
$T = 1/f_{ripple}$

Figure 4-8 compares the output voltage and the charging current. The ripple content has been exaggerated in order to simplify understanding.

It can be shown that the peak current that occurs during the portion of the cycle that the capacitor is charging can be determined from*

$$I_{peak\ diode} \approx I_{dc\ load}\left(1 + 2\pi\sqrt{\frac{V_{peak}}{V_{ripple\ peak}}}\right)$$

and since

$$\sqrt{\frac{V_{peak}}{V_{ripple\ peak}}} \approx \sqrt{\frac{V_{dc}}{\sqrt{2}V_{rms\ ripple}}} = \sqrt{\frac{1}{\sqrt{2}k_r}}$$

$$I_{peak\ diode} \approx I_{dc\ load}\left(1 + 2\pi\sqrt{\frac{1}{\sqrt{2}k_r}}\right)$$

* J. F. Gibbons, "Semiconductor Electronics," p. 252, McGraw-Hill Book Company, New York, 1966.

FIGURE 4-8 *Waveshapes for the half-wave-rectified supply with capacitor filter. (a) Output voltage; (b) diode current.*